Phase segregation in binary sandpiles on fractal bases

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We have built experimentally and have numerically studied sandpiles on a base having a prefractal perimeter. This type of perimeter induces the formation of quite complex pile shapes characterized by both ridges and valleys. The effects of a fractal base on the phase segregation of a binary granular system have been investigated. Both demixing and self-stratification phenomena have been investigated. It is found that the demixing of binary granular piles is enhanced by the prefractal perimeter character. The concentration profiles are given. This is briefly discussed in terms of length scale selection. [S1063-651X(98)14412-4]

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I. INTRODUCTION

Despite its every day familiarity, sandpiles and granular flows have become paradigmatic systems of complexity in physics [1,2]. Granular matter shows behaviors that are intermediate between those of solids and liquids: powders pack like solids and flow like liquids. Granular flows are non-Newtonian [3].

Fascinating properties of sandpiles are the tail shape of piles [4], the dynamics of avalanches versus the angle of repose of the pile [5], hysteresis properties, distribution of forces inside the piles, compaction [2], etc.

A recent experiment, which has received much attention from the scientific community, is the following [6,7]. When a mixture of grains of different sizes is poured between two vertical slabs, a stratification of the mixture in alternating layers of small and large grains is observed. Additionally, there is a tendency for the large and small grain to segregate in different regions of the pile [7]. This self-stratification has been attributed to the difference existing between the angles of repose for the different species. However, this interior self-stratification or phase segregation observed in a vertical Hele-Shaw cell cannot be visualized from the exterior of a tridimensional sandpile. Also, the flow of falling grains should be judiciously chosen for getting self-stratification [8].

From a theoretical point of view, it has been demonstrated that cellular automata [9–11] are helpful for the understanding of granular piles, even though they remain discrete models. Numerous automaton models have been developed. To describe the case of a single-species sandpile in a two-dimensional geometry, Bouchaud *et al.* [12] have developed a theoretical approach of continuum equations. Very recently, Boutreux and de Gennes [13] have extended this continuum description of sand flows to the case of two species. They have theoretically reproduced some self-stratification.

Very recently, we have proposed changing the boundary conditions [14] of the base rather than varying the nature (sand, salt, rice [15], etc.) of the pile. We have investigated sandpiles on bases with a prefractal perimeter, i.e., a perimeter built with a few self-similar iterations. We have found that the geometry of the base on which the sandpile is constructed is a relevant parameter which can sometimes lead to ''exotic'' pile shapes [14]. Contrary to the classic regular bases used in the literature, a base with a prefractal perimeter made with three iterations [see Fig. 1(b)] leads already to a treelike network of valleys in addition to a complex network of ridges. These networks are found to be also fractal-like objects imposing a logarithmic selection of length scales. Also, some angles of repose are forbidden (or selected) by the prefractal perimeter such that granular binary mixtures lead to segregation on the surface of the pile. This can lead to a spectacular pile such as the one presented in Fig. 1. Such a structure looks like natural erosion patterns which have also been recognized as being fractal objects [16]. One advantage of the fractal-like perimeter is to make demixing spectacular: the phase segregation occurs on the surface of the pile itself, as observed in Fig. 1(a).

The aim of the present paper is both to illustrate and to investigate numerically the phase segregation on fractal bases and also to compare the theoretical results with ''experimental results'' on a qualitative level. The simulations themselves allow us to examine the internal structure of three-dimensional piles, even though such investigations are not easily realized experimentally.

II. DISCRETE MODEL

The model is mainly based on the Head and Rodgers model [11]. The sandpile is built on a tridimensional lattice, where the grains occupy a single cubic lattice site. Two types of grains are considered: ''type 1'' and ''type 2,'' which are represented, respectively, by ''black'' and ''white'' grains in our work. Both pure species are characterized by the repose angles $\tan^{-1}(z_{11})$ and $\tan^{-1}(z_{22})$.

However, in a binary mixture the dynamics of each grain is governed by the local angle of repose which depends on the type of the neighboring grains. We note that each grain repose angle depends on the local composition of the surface. The angle may be different for white species on dark grains and conversely for dark on white grains such that the new parameters z_{12} and z_{21} (for cross-interactions) also have

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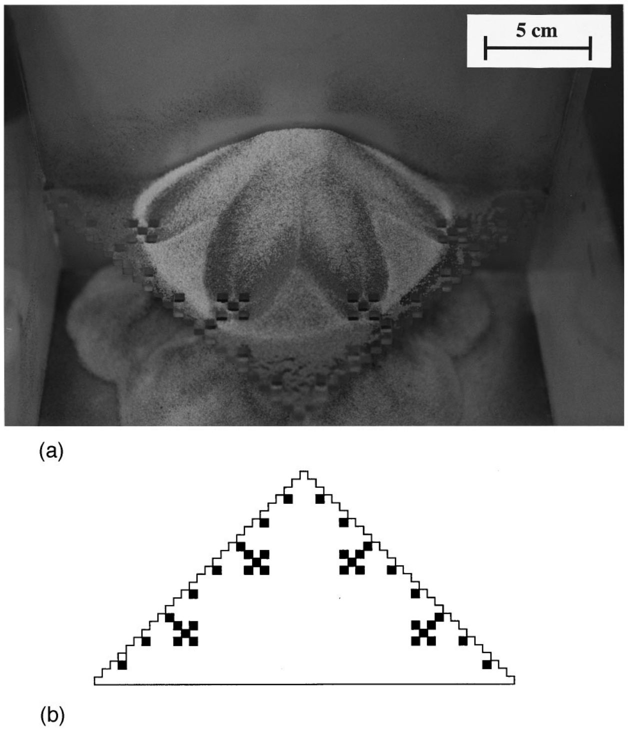

FIG. 1. (a) Binary sandpile on a base with a Koch-like prefractal perimeter of generation 3 ($D_f \approx \ln 5/\ln 3$). A mixture of two kinds of sand grains has been used: (i) diameter of white grains: 0.2–0.3 mm and (ii) diameter of brown grains: 0.07–0.1 mm. Phase segregation (demixing) is clearly observed along the valleys. (b) The fractal perimeter of (a).

to be considered. In full generality, $z_{\alpha\beta}$ corresponds to the maximum slope on which a particle of type α can remain on the top of a particle β without starting to roll down.

The base is made of randomly chosen α and β species with equal probability. At each time step, one white grain (with a probability $\frac{1}{2}$) or one dark grain (with a probability $\frac{1}{2}$) is deposited at the top of the central column of the lattice. The algorithm checks the different angles of all the grains on the surface. When the local angle of repose is larger than the angle $\tan^{-1}(z_{\alpha\beta})$ allowed by the local configuration of the $\alpha\beta$ species, one grain is transfered to one of the first neighboring columns. This new column is chosen randomly among the first neighboring lower columns. This relaxes locally the surface of the pile. The absolute value of the height difference between two neighboring columns is not taken into account for the transfer. No horizontal motion is allowed. This process is iterated over the whole lattice until a stable configuration exists.

Due to the tridimensional character of the setup, the grains follow downward paths on the pile surface such that some sort of ''diffusion'' is ''naturally'' introduced during the pile growth. We stress that a rolling grain cannot climb the pile, implying that the path of a grain is nearly a directed self-avoiding walk. At the border of the base, the falling grains disappear from the structure, i.e., are absorbed beyond the boundaries. The simulation is stopped when the sandpile reaches the border of the base such that the pile shape is nearly invariant. Figure 2 presents a tridimensional sketch of the top of the pile where a white grain is deposited and ''falls'' down until it reaches a locally stable configuration.

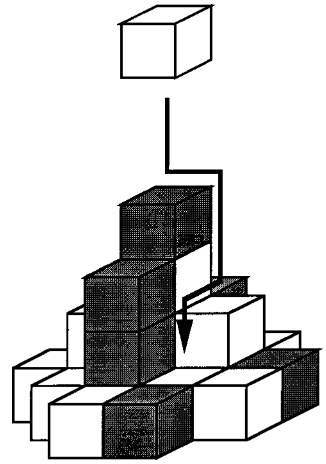

FIG. 2. Tridimensionnal sketch of the rule for the present cellular automaton model. The white grain is deposited at the top of the pile and rolls down following the relaxation rule given in the text and illustrated by the arrow. The parameters are chosen herein to be $z_{11}=z_{12}=2$ and $z_{21}=z_{22}=3$.

III. NUMERICAL RESULTS

In addition to the structure of the base, the present model considers only four parameters: z_{11}, z_{12}, z_{21}, and z_{22}. Herein, we consider integer values of the latter parameters varying from 2 to 5. Thus, 256 distinct situations can be investigated. Head and Rodgers [11] have investigated the $d=2$ version of the model and have distinguished four regimes in the two-dimensional phase diagram $(z_{12}-z_{22}, z_{21}-z_{11})$, assuming without loss of generality that $z_{11}>z_{22}$. Only demixing and self-stratification regimes will be investigated here because of their experimental relevance.

A. Demixing

Figure 3 presents two typical sandpiles obtained numerically on the base illustrated in Fig. 1(b) and on a square base, respectively. The former would have a fractal dimension $D_f=\ln 5/\ln 3$ if the iteration procedure were pursued. The figure presents both a vertical slice accross the pile and a top view of the pile for a particular set of parameters: $z_{11}=5$, $z_{12}=4$, $z_{21}=3$, and $z_{22}=2$. A phase segregation (demixing) of the two species is clearly obtained. This is clearly seen in the transverse view of the pile [Figs. 3(a) and 3(b)]. The white species (type 2) which has a smaller angle of repose is mainly located on the base, while the dark species (type 1) resides at the top of the pile. The fraction of white species contained in the pile is smaller than that for the dark species. This is due to the difference of the repose angles only.

One observes in Figs. 3(a) and 3(b) that the white species forms a triangular structure inside the pyramidal pile. However, the ''head'' of this triangle seems to be broken in the case of the prefractal perimeter [Fig. 3(b)]. This is certainly due to the presence of ''fjords'' in the base at the proximity of this internal structure.

Contrary to piles built on square bases, the demixing is clearly observed in the top views of the pile built on prefractal bases [Fig. 3(c)]. Along the prefractal perimeter, the white species is observed near the holes of the base. A large part of the base is dominated by the white species. This is different

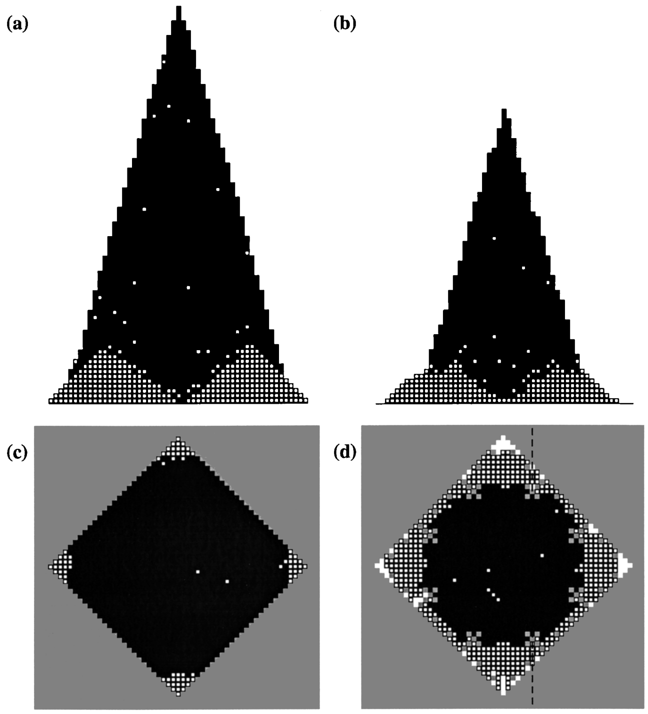

FIG. 3. Typical configuration of the pile for $z_{11}=5$, $z_{12}=4$, $z_{21}=3$, and $z_{22}=2$. (a) Vertical slice of the pile on a 53×53 square base; (b) vertical slice of the pile on a 53×53 base with a prefractal perimeter; (c) top view of the pile of (a); (d) top view of the pile of (b).

from the classical square base for which the white species is only observed at the corners of the square [Fig. 3(c)]. From the top view of the pile of Fig. 3, the white species represents about $\frac{1}{5}$ of the pile on a square base while it represents about $\frac{1}{3}$ of the pile on a base with a prefractal perimeter, although the white and black species are *a priori* deposited at the top of the pile with equal probabiliy.

Moreover, interesting parts of the prefractal base far from the center are not reached by the sandpile, in contrast to the square base case, for which the whole perimeter is reached by the sand if the simulation is pursued long enough. As a corollary, the height of the pile with a prefractal perimeter is smaller than that of the pile built on a square base. The slope of the pile is also modified: with a prefractal perimeter, the slope is smaller than the square base case. Of course, the lower part of the sandpile is more affected than the upper part.

Figure 4 presents other transverse views of the piles as in Fig. 3. The transverse views are taken parallel to the previous ones and passing through holes of the prefractal base (see the dashed line in Fig. 3). The formation of valleys is markedly observed for the prefractal perimeter case. Arrows indicate the presence of valleys on the sides of the sandpiles of Fig. 4.

B. Self-stratification

Figure 5 presents two typical sandpiles obtained numerically on the same prefractal base illustrated in Fig. 1(b) and on a square base, respectively. The figure presents both a vertical slice accross the pile and a top view of the pile. For this particular set of parameters ($z_{11}=3$, $z_{12}=5$, $z_{21}=2$, and $z_{22}=4$), an oblique self-stratification is obtained [Figs. 5(a)–5(c)].

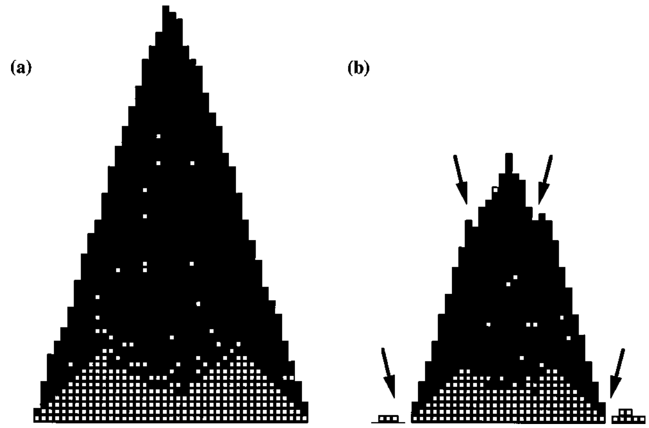

FIG. 4. Shifted transverse view of the piles of Fig. 3. This vertical cut is taken on the dashed line of Fig. 3. Arrows indicate the position of valleys in the case of the prefractal perimeter.

As observed in the transverse view, the self-stratification is better marked here on a square base [Fig. 5(a)] than on a prefractal base [Fig. 5(b)]. We tested the self-stratification on a 161×161 base with a prefractal perimeter of generation 4. Figure 6 presents a transverse view of such a typical pile. The perturbation of the self-stratification is primarily due to the fact that piles built on the prefractal base are smaller than those built on regular bases.

As for piles built on square bases, the self-stratification is not markedly observed for the top view of the pile built on prefractal bases. For a prefractal perimeter, white species (type 2) piling is, however, observed near the fjords of the base. This is quite a different situation from that of the classical square base. Moreover, some parts of the prefractal base far from the center are not reached by the sandpile spreading. This is never the case of the square base.

Figure 7 shows a transverse view of the piles of Fig. 5 shifted such that the vertical cut goes through some holes of

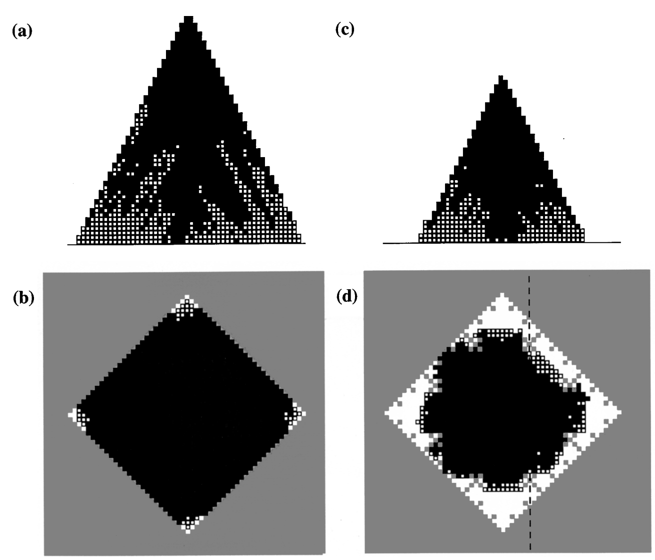

FIG. 5. Typical configuration of the pile for $z_{11}=3$, $z_{12}=5$, $z_{21}=2$, and $z_{22}=4$. (a) Vertical slice of the pile on a 53×53 square base; (b) vertical slice of the pile on a 53×53 base with a prefractal perimeter; (c) top view of the pile of (a); (d) top view of the pile of (b).

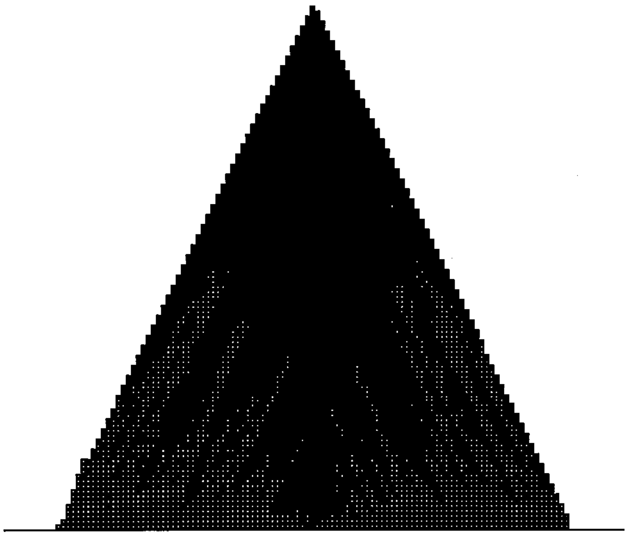

FIG. 6. Vertical transverse view of a typical pile on a 161×161 base.

the fractal base (see the dashed line in Fig. 5). Valley occurences are markedly observed as for the demixing case and are indicated by arrows in Fig. 7.

IV. DISCUSSION

The study of binary sandpiles on bases having a prefractal perimeter leads to amazing effects. We have seen above that the demixing seems to be enhanced on prefractal bases but the self-stratification seems to be slightly disordered by the ''fractality'' of the base. In this section, we show how to understand how these effects occur.

As underlined in our previous experimental work on prefractal bases [14], the formation of branched valleys on the piles leads to the selection of specific length scales [18]. By extension, specific high angles are excluded locally along the pile, especially in the lower part of the sandpile. We have observed these valleys and specific angles on the vertical view of the piles shifted from the center of the pile (see Figs. 4 and 7).

Due to the formation of valleys in which avalanches [17] are mainly dissipated, the pile cannot reach all parts of the irregular base. The fraction of the base reached by sand grains is reduced as the angles $\tan(z_{\alpha\beta})^{-1}$ increase. This is similar to a screening effect. Thus, one understands that for demixing, one species has to reach the farthest parts of the prefractal base since they have the smallest angle of repose,

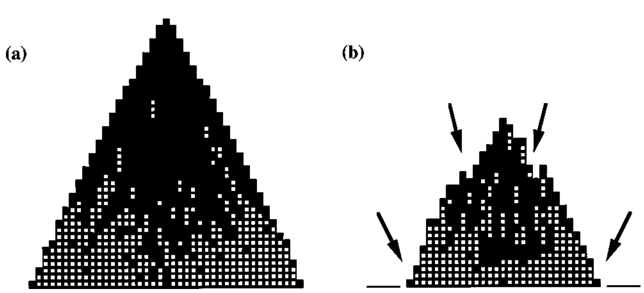

FIG. 7. Shifted view of the piles of Fig. 5. This vertical cut is taken on the dashed line of Fig. 5. Arrows indicate the position of valleys in the case of the prefractal perimeter.

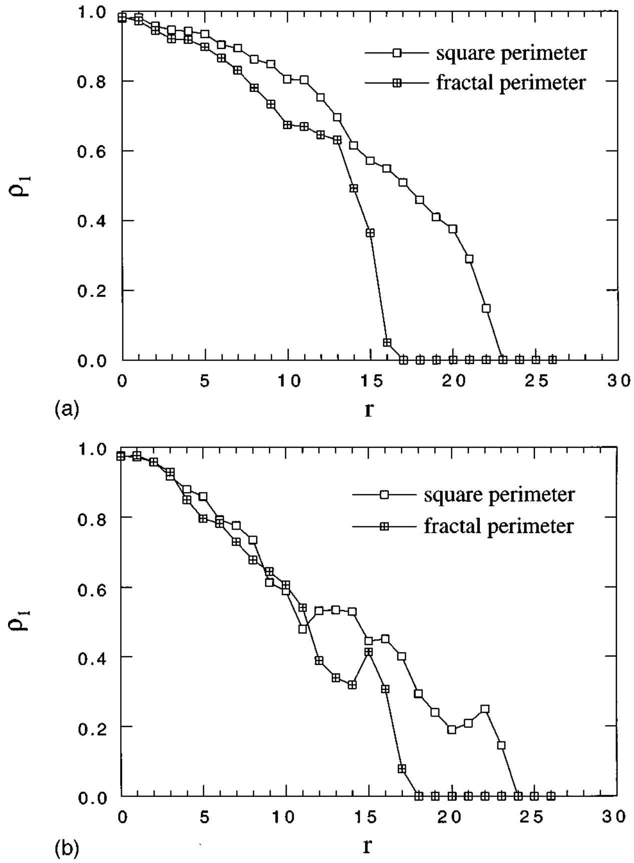

FIG. 8. The normalized concentration profile ρ_1 of the species 1 as a function of the distance r from the center of the base. Both square and prefractal perimeters are given: (a) Demixing situation ($z_{11}=5$, $z_{12}=4$, $z_{21}=3$, and $z_{22}=2$); (b) self-stratification ($z_{11}=3$, $z_{12}=5$, $z_{21}=2$, and $z_{22}=4$).

while the black species having a high angle of repose remains at the center of the base. The more tortuous is the perimeter, the more enhanced is the demixing, and vice versa. This effect should certainly find some application in industry.

The self-stratification phenomenon occurs when the parameters z_{11} and z_{22} are not very different from each other such that one species cannot ''dominate'' the second one. The self-stratification is not destroyed by the fractal perimeter. Due to the formation of valleys, the pile height is, however, always smaller than in the case of the regular base. Also, the position of type 1 and type 2 strates may change with the perimeter structure since they grow from the boundaries.

In order to illustrate the above discussion, Fig. 8 presents the normalized concentration profile ρ_1 of the species 1 as a function of the position r from the center of the pile. Both demixing [Fig. 8(a)] and self-stratification [Fig. 8(b)] cases are shown. Concentration profiles result from averages over five simulations. The size of both prefractal and square bases is 53×53. As discussed above, the concentration profiles are markedly different for the demixing cases with square or prefractal perimeters.

The fraction of type 1 species after the pile reached the perimeter is drastically reduced with a prefractal perimeter. Indeed, we have observed that the fraction of type 1 (black) species reaches about 71% with a fractal perimeter instead of about 78% with a square base. These values may change when the iteration of the prefractal is pursued.

Self-stratification is clearly observed in the concentration profiles of simulation results. Periodiclike damping oscillations are seen in Fig. 8(b). The presence of these oscillations is not affected by a change of the base since the self-stratification is an intrinsic feature of binary sand. However, the respective positions of the maxima and the minima of the oscillations do change. The fraction of species 1 is about 55% in contrast to the demixing case.

V. CONCLUSION

In summary, we have investigated binary sandpiles experimentally and numerically on a base with a prefractal perimeter. The shape of the sandpile exhibits a hierarchical structure of valleys and ridges. Phase segregation has been found to be spectacular in the prefractal geometry, especially the demixing. The relative repose angles seem to be the fundamental parameters since they do or do not allow the pile to map the whole fractal shape through length scale selection.

The fraction of grain species composing the pile is affected by the irregularities in the outer boundary base. The concentration profiles have been investigated and are different on regular and on prefractal bases. This suggests new investigations towards applications of both demixing and fractals.

Our work also suggests new directions of investigations. Instead of varying the nature of the pile (salt, rice [15], etc.), to change the conditions on the boundaries where avalanches are dissipated should receive further attention. Other bases should also be investigated since the demixing situation with a prefractal perimeter leads to a drastic change of the total fraction of species.

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